# Biased News and Voter Turnout

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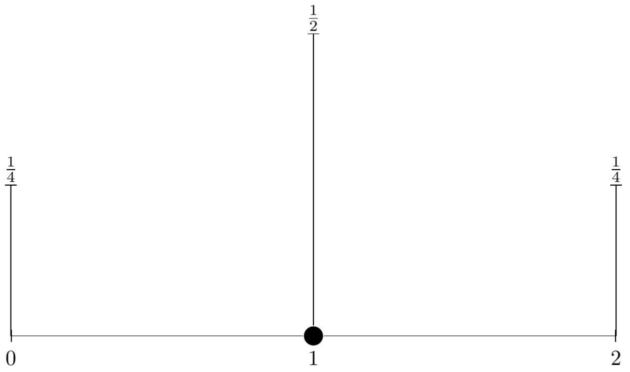
# Why (and How) Does Disinformation Work?

- Traditional models of rational players do not allow for lies
- If receiver fully knows how information is biased, would simply disregard it
- However, this requires fully knowing the details of bias in the source
- Can also adjust for bias with knowledge of the "average" level of bias

# Information Design

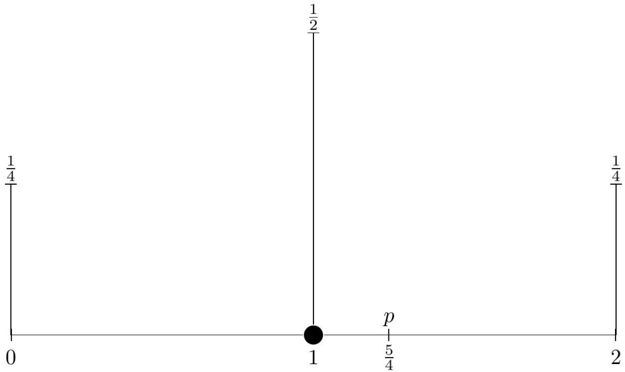
- The study of using information to change incentives (by changing beliefs)
- Sometimes called persuasion
- Seminal paper titled "Bayesian Persuasion" (Kamenica & Gentzkow 2011)
- So far focussed mostly on strategic (but truthful) information

# Buyer-Seller Example: Prior Beliefs




- Three possible states (qualities): $Y \in \{0, 1, 2\}$
- $\mathbb{P}[Y = 0] = \frac{1}{4}; \mathbb{P}[Y = 1] = \frac{1}{2}; \mathbb{P}[Y = 2] = \frac{1}{4}$
- The prior $\mathbb{E}[Y] = 1$

# The Decision Problem

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- $\mathbb{P}[Y = 0] = \frac{1}{4}; \mathbb{P}[Y = 1] = \frac{1}{2}; \mathbb{P}[Y = 2] = \frac{1}{4}$
- The prior $\mathbb{E}[Y] = 1$
- Buy if updated $\mathbb{E}[Y] \geq p = \frac{5}{4}$

# Unbiased Information

- Three possible messages: $X \in \{B, M, G\}$
- The accuracy (informativeness) of the messages are captured by parameter $\theta \in [0, 1]$

$$\begin{aligned}
\mathbb{P}[X = B | Y = 0] &= \frac{1 + 2\theta}{3} \\
\mathbb{P}[X = M | Y = 0] &= \frac{1 - \theta}{3} \\
\mathbb{P}[X = G | Y = 0] &= \frac{1 - \theta}{3}
\end{aligned}$$

- Analogous probabilities for $X = M | Y = 1$ and $X = G | Y = 2$
- Notice that $\theta = 0$ means no information, $\theta = 1$ means perfect information

# Unbiased Information

- Three possible messages: $X \in \{B, M, G\}$
- For this example we just assume $\theta = \frac{4}{10} = 0.4$

$$\begin{aligned}
\mathbb{P}[X = B | Y = 0] &= \frac{1 + 2\theta}{3} = \frac{6}{10} = 60\% \\
\mathbb{P}[X = M | Y = 0] &= \frac{1 - \theta}{3} = \frac{2}{10} = 20\% \\
\mathbb{P}[X = G | Y = 0] &= \frac{1 - \theta}{3} = \frac{2}{10} = 20\%
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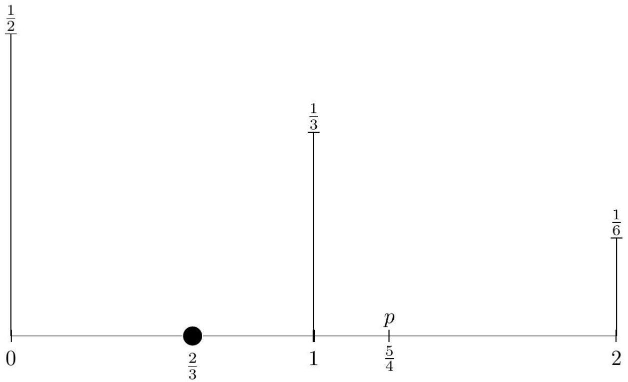
- Analogous probabilities for $X = M|Y = 1$ and $X = G|Y = 2$
- *Importantly*, this determines the frequency of the messages (consistent with prior beliefs):

$$\mathbb{P}[X = B] = 30\%$$
$$\mathbb{P}[X = M] = 40\%$$
$$\mathbb{P}[X = G] = 30\%$$

# Posterior Beliefs (Unbiased)

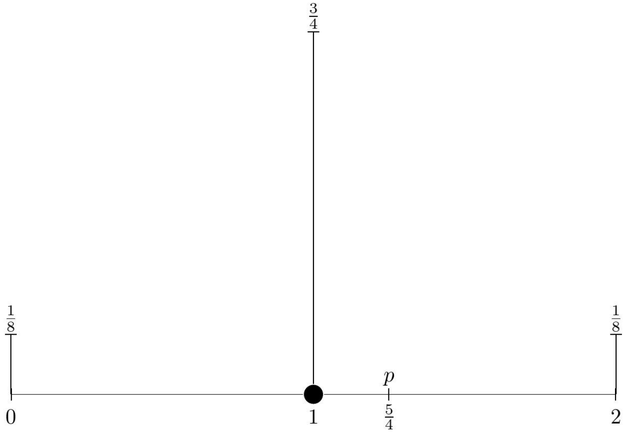
When the message is $X = B$ (30% of the time):




- $\mathbb{E}[Y|X = B] = \frac{2}{3} < p$, so buyer does not buy

# Posterior Beliefs (Unbiased)

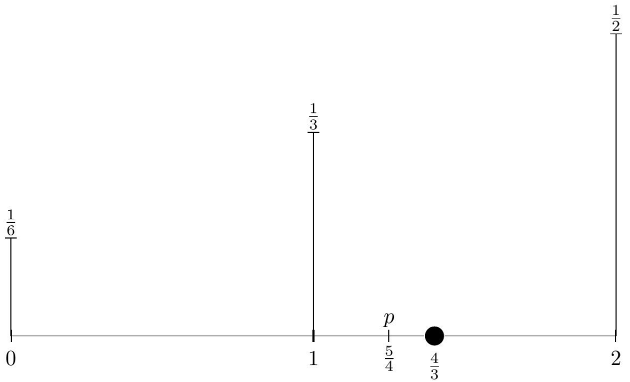
When the message is $X = M$ (40% of the time):




- $\mathbb{E}[Y|X = M] = 1 < p$, so buyer does not buy

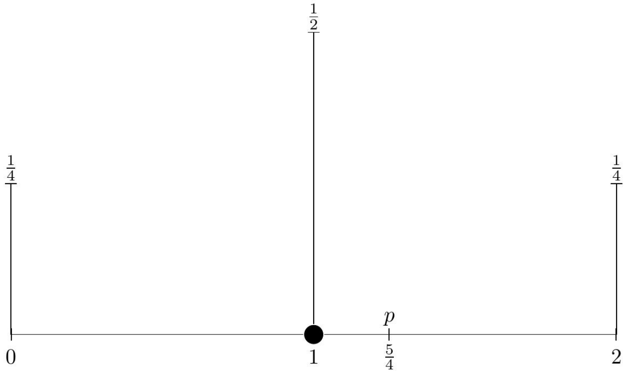
# Posterior Beliefs (Unbiased)

When the message is $X = G$ (30% of the time):




- $\mathbb{E}[Y|X = G] = \frac{4}{3} > p$, so buyer buys only when $X = G$

# Bayes Consistency




- On average, the posterior beliefs should equal the prior (unbiasedness)
- This is always true for correct Bayesian updating
- In our example,
$\mathbb{E}_X\left[\mathbb{E}\left[Y|X\right]\right] = 0.3\left(\frac{2}{3}\right) + 0.4\left(1\right) + 0.3\left(\frac{4}{3}\right) = 1 = \mathbb{E}\left[Y\right]$

# Biased Information

- Again, three possible messages: $X^b \in \{B, M, G\}$
- But this time, regardless of actual $Y$, messages are biased in favor of $X^b = G$

$$\begin{aligned} \mathbb{P}\left[X^b = G|Y=2\right] &= \frac{6}{10} + b \\ \mathbb{P}\left[X^b = B|Y=2\right] &= \frac{2}{10} - \frac{b}{2} \\ \mathbb{P}\left[X^b = M|Y=2\right] &= \frac{2}{10} - \frac{b}{2} \end{aligned}$$

- Similarly, $\mathbb{P}\left[X^b = G|Y=0\right] = \frac{2}{10} + b$; $\mathbb{P}\left[X^b = G|Y=1\right] = \frac{2}{10} + b$
- And $\mathbb{P}\left[X^b = B|Y=0\right] = \frac{6}{10} - \frac{b}{2}$; $\mathbb{P}\left[X^b = M|Y=1\right] = \frac{6}{10} - \frac{b}{2}$

# Biased Information

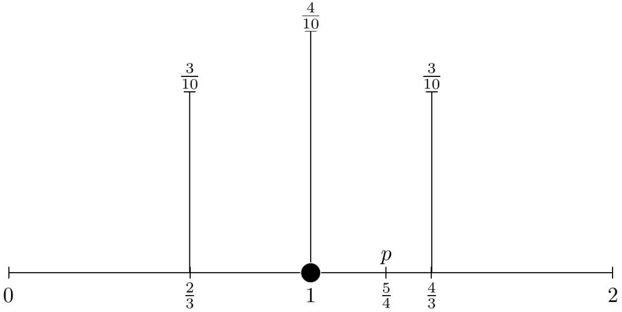
- For this example, we choose $b = \frac{1}{6}$
- Bias distorts the frequency of the three messages:
- Unbiased:

$$\begin{aligned}\mathbb{P}[X = B] &= 30\% \\ \mathbb{P}[X = M] &= 40\% \\ \mathbb{P}[X = G] &= 30\%\end{aligned}$$

- Biased with $b = \frac{1}{6}$:

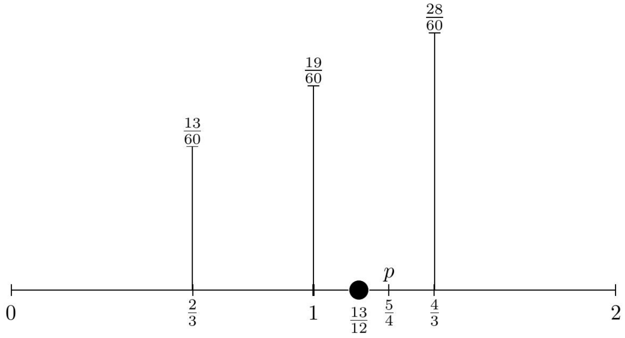
$$\begin{aligned}\mathbb{P}[X = B] &= \frac{13}{60} \approx 22\% \\ \mathbb{P}[X = M] &= \frac{19}{60} \approx 31\% \\ \mathbb{P}[X = G] &= \frac{28}{60} \approx 47\%\end{aligned}$$

# Distribution of Posterior Expectation (Unbiased)




- For unbiased information,
  - 30% of the time, $X = B$ and $\mathbb{E}[Y|X = B] = \frac{2}{3} < p$
  - 40% of the time, $X = M$ and $\mathbb{E}[Y|X = M] = 1 < p$
  - 30% of the time, $X = G$ and $\mathbb{E}[Y|X = G] = \frac{4}{3} > p$

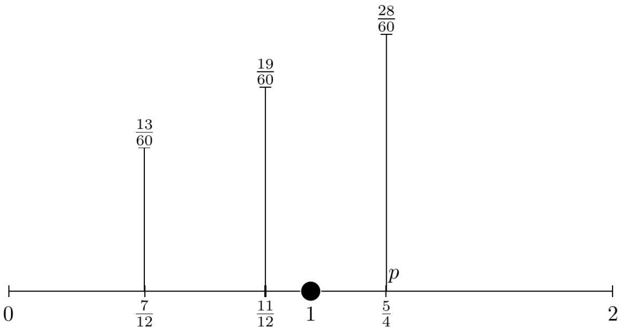
# Distribution of Posterior Expectation (Biased)




- For biased information (naively accepted),

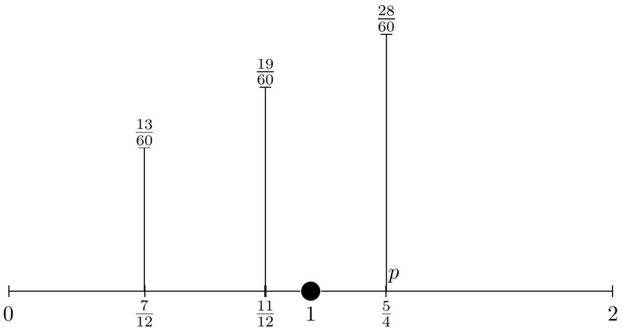
$$
\begin{aligned}
\mathbb{E}_{X^b}\left[\hat{\mathbb{E}}\left[Y|X^b\right]\right] &= \frac{13}{60}\left(\frac{2}{3}\right) + \frac{19}{60}(1) + \frac{28}{60}\left(\frac{4}{3}\right) \\
&= \frac{13}{12} = 1 + \frac{1}{12}
\end{aligned}
$$

# Distribution of Posterior Expectation (Adjusted)



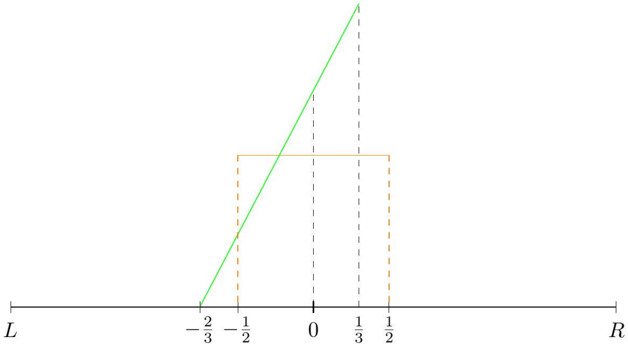

- The buyer adjusts for the bias by shifting posterior beliefs
- Adjusted $\tilde{\mathbb{E}}\left[Y|X^b = G\right] = \frac{4}{3} - \frac{1}{12} = \frac{5}{4}$, and similarly for $X^b = B$ and $X^b = G$
- Now, about 47% of the time, $X^b = G$ and $\tilde{\mathbb{E}}\left[Y|X^b = G\right] = p$
- The sale happens 47% of the time as opposed to 30%

# Distribution of Posterior Expectation (Adjusted)




- The sale happens 47% of the time as opposed to 30%
- Now notice that we chose the bias level $b = \frac{1}{6}$ very carefully
- If $b > \frac{1}{6}$ then even after seeing $X^b = G$, $\mathbb{E}\left[Y|X^b = G\right] < p$, and the buyer *never buys*.
- This suggests a tradeoff for introducing bias.

# Voter Turnout Example




- Voters choose a party based on their perceived (moral/psychological) benefit, and cost, of voting
- Consider a uniform and a triangular distribution of voters
- $f_U(x) = 1$ over $\left[-\frac{1}{2}, \frac{1}{2}\right]$, and $f_T(x) = 2\left(x + \frac{2}{3}\right)$ over $\left[-\frac{2}{3}, \frac{1}{3}\right]$
- With the uniform (unbiased) distribution, mass of $\frac{1}{2}$ votes for the $R$ party, whereas with the triangular distribution, $\frac{5}{9} > \frac{1}{2}$ votes for the $R$ party.
- Hence, bias works in favor of the $R$ party

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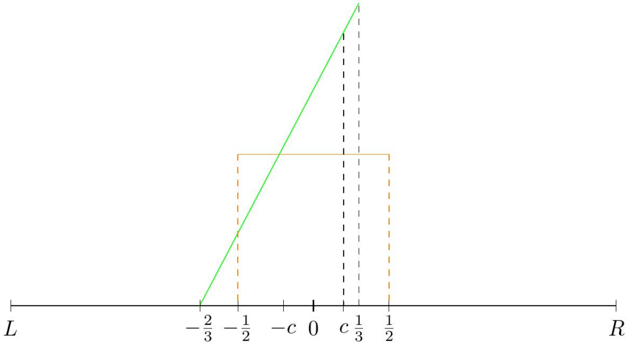


- Consider a uniform and a triangular distribution of voters
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- Now consider a voter with cost $c$ of voting, so only votes if perceived benefit is greater than $c$.
- When $c$ is high enough (threshold value of 0.122), bias hurts the $R$ party, and benefits the $L$ party.